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One residue to rule them all: Electroweak symmetry breaking, inflation and field-space geometry

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ABSTRACT

We point out that the successful generation of the electroweak scale via gravitational instanton configurations in certain scalar-tensor theories can be viewed as the aftermath of a simple requirement: the existence of a quadratic pole with a sufficiently small residue in the Einstein-frame kinetic term for the Higgs field. In some cases, the inflationary dynamics may also be controlled by this residue and therefore related to the Fermi-to-Planck mass ratio, up to possible uncertainties associated with the instanton regularization. We present here a unified framework for this hierarchy generation mechanism, showing that the aforementioned residue can be associated with the curvature of the Einstein-frame target manifold in models displaying spontaneous breaking of dilatations. Our findings are illustrated through examples previously considered in the literature.

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1. Introduction and summary

The seminal discovery of the Higgs field at the LHC has left us with a perfect Standard Model (SM) of particle physics potentially valid up to energies well above the Planck scale $M_P = 2.48 \times 10^{18}$ GeV. At the same time, it left unsolved one of the most mysterious puzzles in particle physics: the so-called hierarchy problem.¹ This has two facets. The first one is the extreme sensitivity of the Higgs mass to whatever happens above the electroweak scale. Several ways of overpassing this difficulty have been proposed in the literature. One of them is to require new physics to appear around the TeV scale (e.g. low-energy supersymmetry, technicolor/composite Higgs, large extra dimensions, see for instance Refs. [2–4]). Another option is to postulate a dynamical relaxation mechanism, like the cosmological attractor scenario [5–8] or its recent variants and generalizations [9,10]. Alternatively, one could require the absence of additional particle states all the way up till the Planck scale [11–16]. Of course, the long-standing question of what happens around and beyond that point still remains. A priori, it is con-

ceivable that quantum gravity corrections may either turn out to be negligibly small, or take care of the problem completely [17,18]. In addition, it might be the case that the fundamental gravitational degrees of freedom above M_P are black holes [19], being their influence on low-energy physics exponentially suppressed at least by a Boltzmann factor proportional to the entropy [20]. We will content here with assuming that, if such contributions are present to start with, the theory is liberated from them in one way or another. This leaves us with the second facet of the hierarchy problem: the origin of the 16 orders of magnitude difference between the electroweak and the Planck scale.

Non-perturbative effects constitute a natural tool for obtaining “small numbers,” especially in models with negligible perturbative corrections. This possibility has been advocated in certain scalar-tensor theories [21] and generalized to scale-invariant models where the Planck mass is generated by the spontaneous breaking of dilatations [22,23], showing explicitly that a second scale can be dynamically generated by an instanton configuration. This idea was recently extended to the Palatini formulation of gravity [24].

In this short paper we generalize the findings of Refs. [21–24], isolating the fundamental ingredients for successfully generating the electroweak scale via instanton effects. In particular, we argue that:

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1. Any scenario able to bring the (conformal) SM scalar sector at large Higgs values to the approximate form

$$\frac{\mathcal{L}}{\sqrt{g}} \approx \frac{M_P^2}{2} R - \frac{1}{2} \frac{M_P^2}{|\kappa_c|} \frac{(\partial\Theta)^2}{\Theta^2} - V_0,$$

with g the metric determinant, R the scalar curvature, $\Theta \propto h^{-1}$, h the Higgs field in the unitary gauge and V_0 an approximately constant potential, will be able to generate a large hierarchy among the electroweak and the Planck scale for sufficiently large values of the inverse residue $|\kappa_c|$.

2. Provided that the scale V_0 is compatible with the COBE normalization [25], the inverse residue $|\kappa_c|$ controls also the inflationary observables. Consequently, if the above splitting mechanism is operative, inflation is intimately related to the electroweak symmetry breaking, making a priori possible to infer the value of the Fermi scale from CMB observations [24,26].
3. The above reasoning holds true irrespectively of the nature of the gravitational interaction (metric, Palatini, Einstein-Cartan ...). The difference boils down to the pole structure of the Higgs kinetic term in the large field regime and, in particular, to the value of the inverse residue $|\kappa_c|$.
4. When single-field models involving the Higgs field are embedded into a fully scale-invariant two-field framework, $|\kappa_c|$ becomes the curvature of the target manifold at large field values. All previous considerations continue to apply for a large class of models displaying a maximally symmetric Einstein-frame kinetic sector. However, unlike the single-field case, the requirement of scale/conformal symmetry is not enough for the successful implementation of the proposed hierarchy generation in biscalar theories. It is also important that no mass term for the Higgs field is (classically) generated by the spontaneous breaking of scale invariance.

Having identified the essence of the mechanism leading to the generation of well separated scales, the approach presented here opens up a new avenue for model building by rephrasing the usual hierarchy problem as a question about the field-space geometry. Among other applications, this could lead to interesting synergies with α -attractors [27–31] and superconformal field theories [32–35].

2. Higgs' pole structure

The SM Lagrangian acquires (classical) conformal invariance when the electroweak scale is set to zero. As can be explicitly seen in the minimal subtraction scheme [36], this implies that no counterterm is needed to renormalize the Higgs mass, which becomes then computable in terms of other SM parameters [37–39]. Interestingly enough, this appealing property remains true even in the presence on a non-minimal coupling to gravity [24]. Having this in mind, we consider the following non-trivial extension of the conformal SM Higgs sector,

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2 + \xi_h h^2}{2} g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2} (\partial h)^2 - \frac{\lambda}{4} h^4. \quad (1)$$

Here $\xi_h > 0$ controls the strength of the Higgs coupling to gravity and λ is the field's quartic self-interaction. Note that for the sake of generality, we have not identified the connection determining the Ricci tensor $R_{\mu\nu}(\Gamma)$ with the Levi-Civita one. Nevertheless, we will assume it to be symmetric ($\Gamma_{\beta\gamma}^\alpha = \Gamma_{\gamma\beta}^\alpha$) in what follows, such that the considered set of theories are torsionless (for non-vanishing torsion scenarios, see e.g. Ref. [40]).

In order to simplify the analysis, it is convenient to get rid of the non-minimal coupling to gravity by moving to the so-called

Einstein frame. This is achieved by considering a Weyl rescaling of the metric $g_{\mu\nu} \rightarrow \omega^2 g_{\mu\nu}$, with conformal factor $\omega^2 = (M_P^2 + \xi_h h^2)/M_P^2$. After some trivial algebra, we get

$$\frac{\mathcal{L}}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{1}{2} \gamma(h) (\partial h)^2 - \frac{\lambda M_P^4 h^4}{4(M_P^2 + \xi_h h^2)^2}, \quad (2)$$

with

$$\gamma(h) = \frac{M_P^2}{M_P^2 + \xi_h h^2} \left(1 + \frac{6\alpha \xi_h^2 h^2}{M_P^2 + \xi_h h^2} \right), \quad (3)$$

and $\alpha = 0$ or 1 for the Palatini or metric formulations, respectively. The essential effect of the Weyl transformation is to transfer the non-linearities associated with the Higgs non-minimal coupling to the scalar sector of the theory. While in the Palatini formulation the connection—and consequently the Ricci tensor—is inert under Weyl rescalings, this is not the case in the metric scenario, where the dependence of the Levi-Civita connection on the metric leads to an additional contribution in Eq. (3). Introducing a variable $\Theta = M_P / \sqrt{M_P^2 + \xi_h h^2}$, we find that for field values relevant for inflation ($h \gg M_P / \sqrt{\xi_h}$, or equivalently $\Theta \ll 1$),² the Lagrangian (2) can be well approximated by

$$\frac{\mathcal{L}}{\sqrt{g}} \approx \frac{M_P^2}{2} R - \frac{M_P^2}{2|\kappa_c|} \frac{(\partial\Theta)^2}{\Theta^2} - \frac{\lambda M_P^4}{4\xi_h^2} (1 - \Theta^2)^2, \quad (4)$$

with

$$\kappa_c \equiv -\frac{\xi_h}{1 + 6\alpha \xi_h}. \quad (5)$$

The pole structure in the above allows for inflation, while making the inflationary observables almost insensitive to the details of the potential [28,41–43] (for a review, see Ref. [44]). The spectral tilt and tensor-to-scalar ratio

$$n_s \simeq 1 - \frac{2}{N_*}, \quad r = \frac{2}{|\kappa_c| N_*^2}, \quad (6)$$

depend only on the number of e-folds of inflation N_* , dictated by the post-inflationary dynamics [45–48] and the inverse of the residue at the inflationary pole at $\Theta = 0$, namely

$$|\kappa_c| \approx \begin{cases} \xi_h & \text{Palatini,} \\ 1/6 & \text{metric,} \end{cases} \quad (7)$$

where in the last step we have taken into account the well-known restriction $\xi_h \gg 1$ needed to generate the right amplitude of primordial density perturbations [26,48]. Note that this result unifies those in Refs. [49,50], see also Refs. [51–53].

Interestingly, the inverse residue (7) controls the Higgs vacuum expectation value (vev),

$$\langle h \rangle \sim \int \mathcal{D}\varphi h e^{-S_E}, \quad (8)$$

with the path integral taken over all fields (including the metric) and S_E the Euclidean action of the theory. After canonically normalizing the field as $\Theta = \exp(-\sqrt{|\kappa_c|} \theta / M_P)$, this equation becomes roughly

$$\langle h \rangle \sim \frac{M_P}{\sqrt{\xi_h}} \int \mathcal{D}\varphi J e^{-W}, \quad (9)$$

² A field redefinition $\Theta \propto h^{-1}$ is convenient to highlight similarities with the two-field scenarios considered in Section 3, where the inflationary region is restricted to a compact field range.

with J the Jacobian of the transformation and

$$W = S_E - \sqrt{|\kappa_c|} \frac{\theta(0)}{M_P}. \quad (10)$$

Note that without loss of generality, we have taken the (instantaneous and localized) source of the scalar field to be at the origin of coordinates.

Assuming the dominant contribution to Eq. (9) to be determined by the extrema of W after regulating the theory with an appropriate higher-dimensional operator (cf. Appendix for details), the vev in the saddle-point approximation becomes [21–24]

$$\langle h \rangle \sim \frac{M_P}{\sqrt{\xi_h}} e^{-\mathcal{W}(|\kappa_c|)}, \quad (11)$$

with $\mathcal{W}(|\kappa_c|)$ a function of the inverse residue $|\kappa_c|$, whose precise expression is irrelevant for the present discussion. It suffices to point out that it remains finite and that the bigger the inverse residue $|\kappa_c|$, the larger the exponential suppression. An accurate result accounting for the regularization of the gravitational instanton can be obtained by numerically solving the system of equations (27) in the Appendix, as done for instance in Refs. [21–24].

A simple inspection of Eq. (5) reveals that the value of $|\kappa_c|$ in the metric formulation ($\alpha = 1$) is restricted to an $\mathcal{O}(1)$ range, $0 < |\kappa_c| \leq 1/6$, meaning that a satisfactory splitting between the electroweak and Planck scales cannot be obtained in the most “vanilla” version of this scenario. In order to reproduce the observed hierarchy, one must inevitably modify the value of the inverse residue at high energies. This can be done in two ways. The first one is to change explicitly the structure of the kinetic sector, as done for instance in Ref. [21]. The second one is to work directly in the Palatini formulation, where $|\kappa_c| \approx \xi_h$ and $\mathcal{W}(|\kappa_c|) \gg 1$. This change of gravity paradigm translates automatically into a larger exponential suppression [24]. Note, however, that, as seen from the unifying glass advocated here, the distinction between these two approaches loses importance. Indeed, when written in the original frame (1), the metric and Palatini formulations differ only by an asymptotically scale-invariant higher-dimensional operator $-3\xi^2 h^2 (\partial h)^2 / (M_P^2 + \xi_h h^2)$, as those required for the self-consistency of the metric effective theory about its cut-off scale [51,54,55].³ From this point of view, the Palatini formulation could be understood as a particular low-energy truncation of the unknown ultraviolet completion of the metric theory, written in a convenient set of variables. The fact that the different approaches followed in Refs. [21] and [24] were able to generate the required hierarchy illustrates that the precise choice of higher-dimensional operators with up to two derivatives of the field to be included in the metric scenario is actually not relevant. For instance, the inclusion in the initial frame of any asymptotically scale-invariant operator of the form $-3\xi^2 h^n (\partial h)^2 / (M_P^2 + \xi_h h^2)^{n/2}$

³ The non-minimal coupling of the Higgs field to gravity makes this theory perturbatively non-renormalizable, meaning that it should be interpreted as an effective field theory to be complemented by a set of higher-dimensional operators suppressed by a given cutoff scale. As shown in Refs. [54,55], the requirement of having a UV completion respecting the symmetries of the original theory forbids the appearance of dangerous operators potentially spoiling the flatness of the inflationary potential. The minimal set of operators is induced by the theory itself as an infinite tower of counterterms aimed to cancel the loop divergences generated order by order in perturbation theory. As compared to the tree-level action, these operators display a hierarchical structure when written in the Einstein-frame, being exponentially suppressed at large field values. The collective effect of all higher-dimensional operators generated by the tree-level action manifest only as localized threshold effects in the SM renormalization group running [51,55]. Although these are important for determining the precise value of the non-minimal coupling to gravity [51,55], they play no role in the (qualitative) instanton dynamics if the associated residue $|\kappa_c|$ stays sufficiently large. In particular, a change in the self-coupling λ by threshold corrections will translate just into a shift of the inverse residue $|\kappa_c|$.

with even $n \geq 2$ would produce the same effect as the operator differentiating metric and Palatini formulations. The most relevant ingredient, up to uncertainties associated with the instanton regularization [21–24], is the effective value of the inverse residue $|\kappa_c|$ at large field values. Provided this is large, the splitting of scales is guaranteed to take place.

3. The geometrical picture

Having understood the key role of the Higgs inverse residue $|\kappa_c|$ in the inflationary observables and the hierarchy between the electroweak and Planck scales, we will show now that it also has a nice geometrical interpretation.

As a warm up, let us extend the Lagrangian (1) by a dilaton field χ , namely $\mathcal{L}' = \mathcal{L} + \mathcal{L}_\chi$, with $\mathcal{L}_\chi / \sqrt{g} = -\frac{1}{2}(\partial\chi)^2$. After performing the same Weyl rescaling we did in the previous section, the kinetic sector of the theory boils down to a non-linear σ -model,

$$\frac{\mathcal{L}'}{\sqrt{g}} \supset -\frac{1}{2}\gamma(h) \left((\partial h)^2 + (\partial\chi)^2 \right), \quad (12)$$

with Gaussian curvature

$$\mathcal{K} = \frac{1}{2} \frac{\gamma'^2(h) - \gamma(h)\gamma''(h)}{\gamma^3(h)}. \quad (13)$$

Here $\gamma(h)$ is given by Eq. (3) and the primes denote differentiation with respect to h . At large field values, the Gaussian curvature (13) becomes approximately constant and coincides, in Planckian units, with the quantity κ_c in Eq. (5),

$$M_P^2 \mathcal{K} \approx \kappa_c. \quad (14)$$

In other words, the residue of the Higgs kinetic pole is nothing else than the dimensionless curvature of the Einstein-frame kinetic manifold. Being a two-dimensional field space, this quantity completely describes the geometry.

Now that we have established the geometrical meaning of κ_c , we can go further and show that the results found in the single field case hold true also in more general biscalar-tensor theories. Although our considerations will be applicable well beyond a particular scenario, we will focus here on the *Higgs-Dilaton model* as a proof of concept [43,56–62] (for a comprehensive overview, see Ref. [63]). As compared to the previous example, this scale-invariant scenario includes also a positive non-minimal coupling of the χ field to gravity, which effectively replaces the bare Planck mass M_P in Eq. (1), i.e.

$$\frac{\mathcal{L}'}{\sqrt{g}} = \frac{\xi_\chi \chi^2 + \xi_h h^2}{2} g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4}h^4. \quad (15)$$

Some comments are in order at this point. The attentive reader will have probably noticed that we have not included a scale (and conformally) invariant $\alpha\chi^2 h^2$ term, leading to a non-vanishing expectation value for the Higgs field after the spontaneous breaking of scale invariance, i.e. $\langle h \rangle \propto \alpha\langle\chi\rangle$ with the parameter α tuned to a value $\alpha \sim 10^{-34}\xi_\chi$ in order to reproduce the hierarchy of scales and $\xi_\chi \ll 1$ as required by inflationary phenomenology [57]. The absence of this operator is indeed crucial for the successful implementation of the mechanism considered here and can be justified at different levels. First, an explicit Higgs-Dilaton coupling can be excluded by requiring the dilaton field χ to display an exact shift symmetry in the matter sector, broken only mildly by gravitational interactions. Interestingly, this would also forbid the inclusion of a

quartic dilaton self-interaction leading to a cosmological constant term when moving to the Einstein frame. Beyond symmetry restrictions, the presence of a quartic mixing term could make the theory ill-behaved in the ultraviolet domain [42], motivating also its exclusion on the basis of self-consistency. Finally, a situation of this sort might arise naturally in more “exotic” scenarios such as the Fishnet Conformal Field Theory [64], where it is indeed possible to have spontaneous breaking of scale/conformal invariance without generation of masses for the fields, at least in the large- N limit [65].

Provided that the mixing $\alpha \chi^2 h^2$ term is absent, either because of one of the aforementioned reasons or by simply tuning its coefficient to zero, the mass of the Higgs field is classically zero. To guarantee that this is the case even when quantum corrections are taken into account,⁴ the renormalization procedure must preserve the symmetries of the classical theory at all orders of perturbation theory, i.e. the scale invariance of the tree-level action and the approximate shift symmetry of the field χ . In the absence of particle thresholds above the electroweak scale, this can be achieved by employing a “scale-invariant” regularization scheme [13,66,67]. In this setting, the ‘t Hooft-Veltman renormalization point μ in dimensional regularization is replaced by a field dependent renormalization point $\mu(h, \chi)$ accounting for the usual dimension mismatch among bare and renormalized coupling constants. Different choices of μ correspond to different theories. In particular, a change of μ can always be compensated for by the inclusion of a specific set of higher dimensional operators in the tree-level action. For the sake of concreteness, we will assume the quantization to be performed in the non-minimally coupled frame (15), according to the so-called prescription I of Ref. [46], where $\mu^2 \sim \xi_\chi \chi^2 + \xi_h h^2$. Working in this context, the Higgs field is completely desensitized from quantum corrections involving the dilaton. This remains true even when perturbative gravitational corrections are taken into account. In particular, graviton loops generate a contribution to the effective potential of the form $\lambda^2 h^8 / (\xi_\chi \chi^2 + \xi_h h^2)^2$. In other words, provided that the aforementioned assumptions hold, a Higgs mass term cannot be generated at any order of perturbation theory in the $\alpha \rightarrow 0$ limit; see Refs. [13,22,68,69]. The graviscalar instanton discussed in the previous section becomes then a viable option to induce the electroweak scale. The detailed analysis carried out in Ref. [22], revealed that it is indeed possible to generate a non-vanishing Higgs vev in the Higgs-Dilaton model via this mechanism. To illustrate this result in our language, it is again convenient to perform the Weyl transformation $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$ with conformal factor $\Omega^2 = (\xi_\chi \chi^2 + \xi_h h^2) / M_P^2$. This yields the Lagrangian density

$$\frac{\mathcal{L}'}{\sqrt{g}} = \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \gamma_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b - U(\varphi), \quad (16)$$

where we have organized the fields χ and h into a vector $\varphi^a = (\chi, h)$ with $a, b = 1, 2$ and defined the Einstein-frame potential $U(\varphi) \equiv \Omega^{-4} V(\varphi)$. The field-space metric in this expression

$$\gamma_{ab} = \frac{M_P^2}{\xi_\chi \chi^2 + \xi_h h^2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{6\alpha}{\xi_\chi \chi^2 + \xi_h h^2} \begin{pmatrix} \xi_\chi^2 \chi^2 & \xi_\chi \xi_h \chi h \\ \xi_\chi \xi_h \chi h & \xi_h^2 h^2 \end{pmatrix}, \quad (17)$$

is a straightforward generalization of the coefficient $\gamma(h)$ in Eq. (3). As before, the difference between the metric and Palatini approaches is accounted for by the value of α . The kinetic sector in (16) can be made diagonal by introducing the variables [43,61,70]⁵

$$\Theta = |\bar{\kappa}_c| \frac{(1 + 6\alpha \xi_\chi) \chi^2 + (1 + 6\alpha \xi_h) h^2}{\xi_\chi \chi^2 + \xi_h h^2}, \quad (18)$$

$$e^{\frac{2\sqrt{|\bar{\kappa}_c|}\Phi}{M_P}} = \frac{|\kappa_c|}{|\kappa|} \frac{(1 + 6\alpha \xi_\chi) \chi^2 + (1 + 6\alpha \xi_h) h^2}{M_P^2}, \quad (19)$$

with κ_c defined in Eq. (5), and

$$\bar{\kappa}_c = \kappa_c|_{\xi_h \rightarrow \xi_\chi}, \quad \kappa = \kappa_c \left(1 - \frac{\xi_\chi}{\xi_h}\right). \quad (20)$$

In terms of these quantities, the Higgs-Dilaton Lagrangian reads [43,61,70]

$$\frac{\mathcal{L}'}{\sqrt{g}} = \frac{M_P^2}{2} \tilde{R} - \frac{K(\Theta)}{2} (\partial\Theta)^2 - \frac{\Theta}{2} (\partial\Phi)^2 - U(\Theta), \quad (21)$$

with

$$K(\Theta) = \frac{M_P^2}{4\Theta} \left(\frac{1}{|\kappa|(\Theta - \sigma)} + \frac{1}{|\bar{\kappa}|(\Theta - 1)} \right), \quad (22)$$

$$U(\Theta) = \frac{\lambda M_P^4}{4|\bar{\kappa}|^2} (1 - \Theta)^2, \quad (23)$$

and

$$\bar{\kappa} = \kappa|_{\xi_h \rightarrow \xi_\chi}, \quad \sigma = \frac{\bar{\kappa}_c}{\kappa_c}. \quad (24)$$

Note that the kinetic function K in Eq. (22) has poles at $\Theta = 0$, σ and 1. The first two are potentially explored during inflation, with $\sigma \propto \xi_\chi$ encoding the differences with the warm-up example above. The last one is a “Minkowski” pole associated with the ground state of the theory, namely $\langle h \rangle = 0$, $\langle \chi \rangle \approx M_P$, or equivalently, $\langle \Theta \rangle = 1$ and $\langle \Phi \rangle = M_P / (2\sqrt{|\bar{\kappa}_c|}) \log(1/|\kappa|\sigma)$.

A geometrical interpretation of the inverse residues $|\kappa|$ and $|\bar{\kappa}|$ can be obtained from the direct computation of the Gaussian curvature of the target manifold (17), namely

$$\mathcal{K} = \frac{\kappa \bar{\kappa}}{M_P^2} \frac{\kappa(\Theta - \sigma)^2 + \bar{\kappa}(\Theta - 1)^2}{[\kappa(\Theta - \sigma) + \bar{\kappa}(\Theta - 1)]^2}. \quad (25)$$

As clearly illustrated by this expression, $|\kappa|$ and $|\bar{\kappa}|$ coincide with the dimensionless curvature of γ_{ab} around their corresponding poles $\Theta = \sigma$ and $\Theta = 1$.

Focusing on the field values relevant for inflation and restricting ourselves to the phenomenologically allowed limit $\xi_\chi \ll \xi_h$ ($\sigma \ll |\kappa| \simeq |\kappa_c|$) [57], the kinetic function (22) boils down to a quadratic pole structure reminiscent of that found in the single field scenario (4), namely

$$K(\Theta) \approx \frac{M_P^2}{4|\kappa_c|\Theta^2}. \quad (26)$$

⁴ Note that if a non-vanishing mixing term between the Higgs and the dilaton is perturbatively generated, there is no sense in discussing non-perturbative ways of inducing the Fermi scale, since these effects are always subdominant. In such a case, a severe fine-tuning—completely analogous to the situation in the SM—is needed for the electroweak-to-Planck scale hierarchy to be reproduced.

⁵ Note that Θ is restricted to the interval $\sigma \leq \Theta \leq 1$. In addition, we chose an “exponential map” in order to highlight that Φ is the Goldstone boson associated with the non-linear realization of scale symmetry—the dilaton.

The residue of this pole controls again the inflationary observables in the corresponding limit [43,70], as well as the generation of the electroweak scale, cf. Eq. (11) and the Appendix. As in the single field case, the non-perturbative generation of the observed hierarchy $\langle h \rangle \ll M_P$ cannot be achieved in the metric scenario without appropriately modifying the kinetic sector of the theory. This is again not the case in the Palatini formulation, where the large value of the inverse residue $|\kappa_c|$ allows to obtain the desired mass splitting.

The above results can be straightforwardly extended to a more general class of scale-invariant models constructed on the basis of volume-preserving diffeomorphisms [42,71], provided that they display the same ground state and a maximally symmetric Einstein-frame kinetic manifold at large field values. As shown in Refs. [41,61], this additional symmetry of the target manifold restricts the leading kinetic pole structure of such models to be quadratic. Whether the considerations presented in this paper remain valid outside this equivalence class is still an open issue. In particular, if present, higher-order poles should be sufficiently suppressed—in a way similar to Ref. [72]—to guarantee the existence of a source term selecting the graviscalar instanton solution advocated in Refs. [21–24]. Since we do not aim to provide here an exhaustive classification of plausible scenarios, but rather a guiding principle unifying existing models in the literature, we postpone the study of this issue for the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Instanton computation

We present here some details on the graviscalar instanton solution and its relation to the inverse residue. It should be clearly stated that the following analytical estimates should be taken with a grain of salt, being necessary a numerical treatment in order to extract quantitative results. In this regard, we refer the reader to Refs. [21,24].

Single field case—Our starting point is Eq. (10), with the Euclidean action S_E supplemented by a higher-dimensional Einstein-frame operator with the approximate form $\beta(\partial\theta)^6/M_P^8$ in the high-energy regime and taming an unphysical field divergence at the origin.⁶ We emphasize that this term is chosen here for illustration purposes only, being the considerations below also valid for more general higher-order operators [22]. In particular, other higher-dimensional operators involving more than two derivatives could be also considered. The exact form of these terms is determined by the unknown UV completion of the theory and cannot be addressed from the bottom-up approach presented here. Different choices modify the precise way the instanton is regularized at its core, leading to a different dependence of the instanton

action—and consequently of the scale splitting—on the operators' coefficients.

As preferred by the symmetries of point-like sources, we will assume the classical instanton configuration to be $O(4)$ -symmetric, such that the associated Euclidean metric takes the form $ds^2 = f^2(r)dr^2 + r^2 d\Omega_3$, with r the radial component and $d\Omega_3$ the line element of the unit 3-sphere. Neglecting the contributions from the potential and its derivative [21,24], the rr -component of the Einstein equations and the scalar field equation of motion can be written as

$$\begin{aligned} \frac{3 - 3f^2}{\tilde{r}^2} &= \frac{1}{2}(\partial_{\tilde{r}}\tilde{\theta})^2 + \frac{5\beta}{f^4}(\partial_{\tilde{r}}\tilde{\theta})^6, \\ \partial_{\tilde{r}}\tilde{\theta} + \frac{6\beta}{f^4}(\partial_{\tilde{r}}\tilde{\theta})^5 &= -\frac{\sqrt{|\kappa_c|}}{2\pi^2} \frac{f}{\tilde{r}^3}, \end{aligned} \quad (27)$$

with $\tilde{r} = r M_P$ and $\tilde{\theta} = \theta/M_P$ appropriate dimensionless variables. These equations are supplemented by the flat boundary conditions $f(\tilde{r}) \sim 1$, $\tilde{\theta}(\tilde{r}) \sim 0$ at $\tilde{r} \rightarrow \infty$.

We will be mainly interested in the behavior of the system (27) in the core of the instanton, located at distances $\tilde{r} \lesssim \tilde{r}_0 \equiv (\beta|\kappa_c|^2)^{1/12}$, where the higher-dimensional operator $\beta(\partial\theta)^6/M_P^8$ dominates. The reason we focus on this particular regime is because its dynamics is what influences the instanton the most in the considered example, as shown in details in Refs. [21,22]. There, the instanton solution asymptotes, up to order-one numerical contributions, to

$$\begin{aligned} \tilde{\theta} &\sim -\log \tilde{r}_0 \sim -\log(\beta|\kappa_c|^2), \\ f &\sim \tilde{r}_0^{4/5} \beta^{1/10} |\kappa_c|^{-3/10} \sim (\beta^{-1}|\kappa_c|)^{-1/6}. \end{aligned} \quad (28)$$

Using these expressions one can easily show that the Euclidean action evaluates to $S_E \sim \sqrt{|\kappa_c|}$, which in turn translates into a large exponent $\mathcal{W}(|\kappa_c|) \sim \sqrt{|\kappa_c|}(\log(\beta|\kappa_c|^2) - \mathcal{O}(1))$ in Eq. (11) and a small Higgs vev for a sufficiently large inverse residue $|\kappa_c|$.

Instanton in biscalar case—We turn now to the generalization of the above result to the two-field case; see also Ref. [22]. In particular, we are interested in evaluating the action W on the graviscalar instanton solution for the Higgs-Dilaton scenario. The first step is to neglect the low-energy pole at $\Theta = 1$ in Eq. (21), such that the target manifold becomes maximally symmetric. In the limit $\sigma \ll |\kappa| \approx |\kappa_c|$, the “temporal” component of the Einstein equations and the equations of motion for the scalar fields become respectively

$$\begin{aligned} \frac{3 - 3f^2}{\tilde{r}^2} &= \frac{\Theta}{2}(\partial_{\tilde{r}}\tilde{\Phi})^2 + \frac{(\partial_{\tilde{r}}\ln\Theta)^2}{8|\kappa_c|} + \frac{5\beta}{f^4}(\partial_{\tilde{r}}\tilde{\Phi})^6, \\ \Theta \partial_{\tilde{r}}\tilde{\Phi} + \frac{6\beta}{f^4}(\partial_{\tilde{r}}\tilde{\Phi})^5 &= -\frac{\sqrt{|\kappa_c|}}{2\pi^2} \frac{f}{\tilde{r}^3}, \\ \frac{f}{\tilde{r}^3} \partial_{\tilde{r}} \left(\frac{\tilde{r}^3}{f} \partial_{\tilde{r}} \ln \Theta \right) &= 2|\kappa_c| \Theta (\partial_{\tilde{r}}\tilde{\Phi})^2, \end{aligned} \quad (29)$$

where we have again neglected the contributions of the Einstein-frame potential and its derivative and defined the dimensionless variables $\tilde{r} = r M_P$ and $\tilde{\Phi} = \Phi/M_P$.

As in the single-field case, Φ becomes singular inside the instanton core $\tilde{r} \lesssim \tilde{r}_0 \equiv (|\kappa_c|^5 \tilde{\kappa}_c^{-3} |\beta|)^{1/12}$ in the absence of the higher-dimensional operator ($\beta=0$). On the other hand, we can approximate $\Theta \sim \sigma$ there. Finally, $\sqrt{|\kappa_c|}\tilde{\Phi} \sim -\sqrt{|\kappa_c|} \log(|\kappa_c|^5 \tilde{\kappa}_c^{-3} \beta)$.

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⁶ In the frame (1), this operator reads $\beta_0(\partial h)^6/(M_P^2 + \xi_h h^2)^4$ with $\beta_0 = \beta(\xi_h/|\kappa_c|)^3$.

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